Resolution

Examples

Lucky Student

1. Anyone passing his history exams and winning the lottery is happy.

2. Anyone who studies or is lucky can pass all his exams.

3. John did not study but he is lucky.

4. Anyone who is lucky wins the lottery.

Prove that John is happy!

Anyone passing his history exams and winning the lottery is happy.

∀ X (pass (X,history) ∧ win (X,lottery) → happy (X))

Anyone who studies or is lucky can pass all his exams.

∀ X ∀ Y (study (X) ∨ lucky (X) → pass (X,Y))

John did not study but he is lucky.

¬ study (john) ∧ lucky (john)

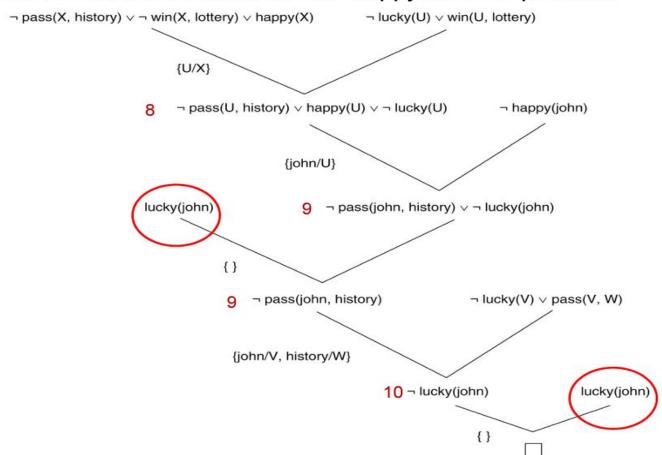Anyone who is lucky wins the lottery.

∀ X (lucky (X) → win (X,lottery))

CNF

1. $\neg$ pass (X, history) $\lor \neg$ win (X, lottery) $\lor$ happy (X)
2. $\neg$ study (Y) $\lor$ pass (Y, Z)
3. $\neg$ lucky (W) $\lor$ pass (W, V)
4. $\neg$ study (john)
5. lucky (john)
6. $\neg$ lucky (U) $\lor$ win (U, lottery)

Into these clauses is entered, in clause form, the negation of the conclusion:

7. $\neg$ happy (john)

One resolution refutation for the "happy student" problem.

¬ pass(X, history) ∨ ¬ win(X, lottery) ∨ happy(X)   ¬ lucky(U) ∨ win(U, lottery)

{U/X}

8   ¬ pass(U, history) ∨ happy(U) ∨ ¬ lucky(U)   ¬ happy(john)

{john/U}

lucky(john)   9   ¬ pass(john, history) ∨ ¬ lucky(john)

{ }

9   ¬ pass(john, history)   ¬ lucky(V) ∨ pass(V, W)

{john/V, history/W}

10 ¬ lucky(john)   lucky(john)

{ }

□

Exciting life

1. All people that are not poor and are smart are happy.

2. Those people that read are not stupid.

3. John can read and is wealthy.

4. Happy people have exiting lives.

Can anyone be found with an exciting life?

All people who are not poor and are smart are happy. Those people who read are not stupid. John can read and is wealthy. Happy people have exciting lives. Can anyone be found with an exciting life?

We assume $\forall X$ (smart $(X) \equiv \neg$ stupid (X)) and $\forall Y$ (wealthy $(Y) \equiv \neg$ poor (Y)), and get:

$\forall X$ ($\neg$ poor $(X) \wedge$ smart $(X) \rightarrow$ happy (X))

$\forall Y$ (read $(Y) \rightarrow$ smart (Y))

read (john) $\wedge \neg$ poor (john)
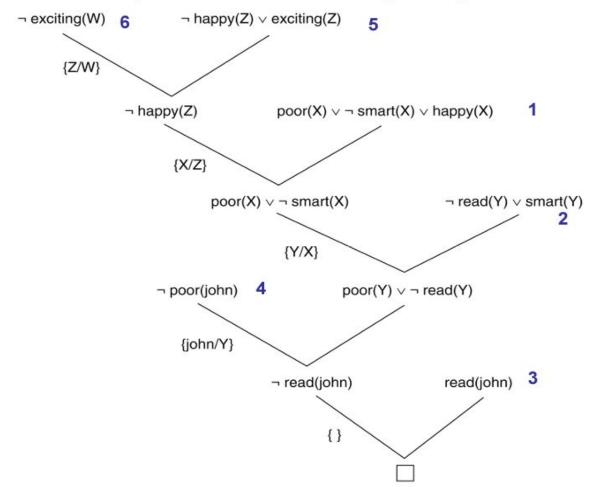
$\forall Z$ (happy $(Z) \rightarrow$ exciting (Z))

The negation of the conclusion is:

$\neg \exists W$ (exciting (W))

These predicate calculus expressions for the "exciting life" problem are transformed into the following clauses:

poor $(X) \vee \neg$ smart $(X) \vee$ happy (X) **1**

$\neg$ read $(Y) \vee$ smart (Y) **2**

read (john)   **3**

$\neg$ poor (john)   **4**

$\neg$ happy $(Z) \vee$ exciting (Z) **5**

$\neg$ exciting (W) **6**

Resolution proof for the "exciting life" problem.

¬ exciting(W) **6**   ¬ happy(Z) ∨ exciting(Z) **5**

{Z/W}

¬ happy(Z)   poor(X) ∨ ¬ smart(X) ∨ happy(X) **1**

{X/Z}

poor(X) ∨ ¬ smart(X)   ¬ read(Y) ∨ smart(Y) **2**

{Y/X}

¬ poor(john) **4**   poor(Y) ∨ ¬ read(Y)

{john/Y}

¬ read(john)   read(john) **3**

{ }

□

- Anything anyone eats and not killed is food.

- Anil eats peanuts and still alive

- Harry eats everything that Anil eats.

- John likes all kind of food.

- Apple and vegetable are food

- Prove by resolution that:

- John likes peanuts.

$\forall$x: food(x) → likes(John, x)

food(Apple) $\wedge$ food(vegetables)

$\forall$x $\forall$y: eats(x, y) $\wedge$ ¬ killed(x) → food(y)

eats (Anil, Peanuts) $\wedge$ alive(Anil).

$\forall$x : eats(Anil, x) → eats(Harry, x)

$\forall$x: ¬ killed(x) → alive(x) ⎤ **added predicates.**

$\forall$x: alive(x) →¬ killed(x) ⎦

likes(John, Peanuts)

∀x ¬ food(x) V likes(John, x)

food(Apple) ∧ food(vegetables)

∀x ∀y ¬ eats(x, y) V killed(x) V food(y)

eats (Anil, Peanuts) ∧ alive(Anil)

∀x ¬ eats(Anil, x) V eats(Harry, x)

∀x ¬killed(x)] V alive(x)

∀x ¬ alive(x) V ¬ killed(x)

likes(John, Peanuts).

¬likes(John, Peanuts)   ¬ food(x) V likes(John, x)

{Peanuts/x}

¬ food(Peanuts)   ¬ eats(y, z) V killed(y) V food(z)

{Peanuts/z}

¬ eats(y, Peanuts) V killed(y)   eats (Anil, Peanuts)

{Anil/y}

Killed(Anil)   ¬ alive(k) V ¬ killed(k)

{Anil/k}

¬ alive(Anil)   alive(Anil)

{   }   **Hence proved.**